HEAP

Time Complexity Analysis of Max-Heap and Heap Sort

Building a Max-Heap

Method 1: Using Heapify

Time Complexity: O(n)

The method buildHeapUsingHeapify invokes the method heapifySubtree for n/2 nodes from the bottom in the array. Each heapifySubtree takes O(log n) time in the worst case, but still, overall time complexity is O(n), as nodes located close to the leaves take O(1) time to perform the heapify operation.

2) Method 2: Adding Elements One by One

Time Complexity: O(n log n)

Approach: For this operation, an element is added to the heap, then it's fixing the property of the heap through moving the added element up the heap. This operation has O(log n) per element time complexity, hence O(n log n) in total.

Heap Sort Algorithm

Building the max-heap using one of the two methods mentioned above.

Repeatedly extracting the maximum element and restoring the heap property.

Time Complexity: O(n log n)

First, building the max-heap is performed in O(n) time for this operation (if the method Heapify is used) or O(n log n) time (in the case of adding elements to the heap one by one).

The second part of extracting the maximum takes O(log n) time, and per extraction, the heap property needs to be restored. This is repeated n - 1 times. Thus, Heap Sort has time complexity O(n log n).

Best, Worst, and Average Case Scenarios:

Best Case:

When all elements of the input array are the same, then in that case, the time complexity of building a max-heap using the Heapify method is also O(n), and it gives a misleading time complexity for the Heap Sort algorithm.

Worst Case: - The worst time complexity required to build the max-heap is O(n log n). Accordingly, the Heap Sort algorithm has a worst-case time complexity of O(n log n) for any method to build the max-heap.

Average Case: - The average-case time complexity for both building the max-heap and the Heap Sort algorithm is O(n log n). So, the Heapify method will be building the max-heap way more efficiently than the "add elements one by one" approach, as it has a time complexity of O(n) against the time complexity of O(n log n) in the other case. But it has to be pointed out that the Heap Sort algorithm has a time complexity in overall of O(n log n) in best, worst, and average cases, irrespective of the method used to build the max-heap.